

# NONLINEAR SLOW-WAVE PROPAGATION ON PERIODIC SCHOTTKY COPLANAR LINES

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## ABSTRACT

Nonlinear wave propagation along periodic Schottky slow-wave structures with voltage dependent capacitance is analytically described by simple wave equations. As an exemplary result of the theory, a special structure is proposed which can be used for traveling-wave second harmonic generation or parametric amplification with high efficiency. This is experimentally verified on a model line.

## INTRODUCTION

In recent years, a growing interest has been paid to the study of slow-wave structures for different applications in MMIC's. In particular, it has been shown that the Schottky coplanar waveguide is an attractive candidate when electronically variable properties are desired. Recently, it has been pointed out that a waveguide on periodically doped semiconductor substrate might be advantageous regarding low losses and large slow-wave factors /1,2/.

Nearly all previous work has been dedicated to the small signal properties of such lines. However, there are several reasons which demand for a general large signal analysis. Due to the expected low dissipation in periodic structures the attenuation per wavelength - the quality factor - has achieved values where the nonlinearity may no longer be neglected. This is especially true regarding high power operation as in modern MESFET distributed amplifiers. On the other hand, the periodic structure yields a relatively high value of the characteristic impedance so that the voltage amplitude is enlarged in comparison with slow-wave structures on homogeneous substrate. Finally, even at frequencies near 10 GHz the guide wavelength may be smaller than 500  $\mu\text{m}$  so that traveling wave concepts should be applied to any nonlinear device with dimensions in excess of 100  $\mu\text{m}$ . Therefore, a general large signal analysis of slow-wave propagation along periodic Schottky coplanar lines is needed.

## PERIODIC SCHOTTKY COPLANAR LINES

Fig. 1 shows a schematic view of the periodic structure as has been proposed recently /2/. Instead of a homogeneous semiconducting surface layer, a periodically doped substrate is used. The central strip conductor is assumed to form a Schottky contact. Thus from another point of view,

the structure of Fig. 1 is that of a chain of planar Schottky diodes periodically coupled by small parts of well-fitting coplanar transmission lines in MMIC technology. In this approach, it is

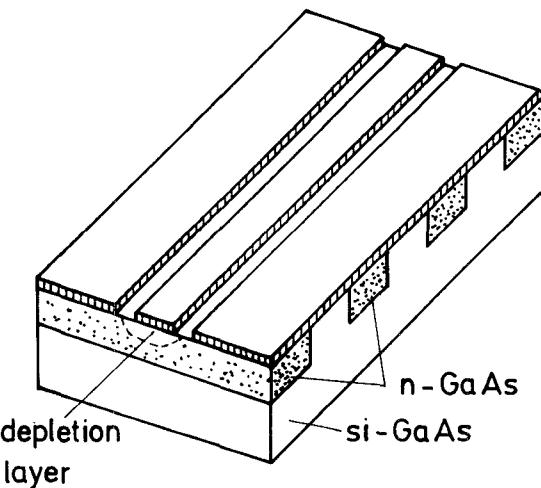


FIG. 1. SCHEMATIC VIEW OF THE PERIODIC SCHOTTKY COPLANAR WAVEGUIDE /2/.

assumed that the length of the Schottky diode, i.e. the length of the doped layers is small as compared to the wavelength on the Schottky coplanar line. As a result, this periodic loading of a coplanar waveguide by Schottky diodes is very similar to the monolithic GaAs traveling-wave amplifier /3/.

## NONLINEAR WAVE PROPAGATION

Provided that the wavelength under consideration is sufficiently larger than the length of the period, the equivalent circuit in Fig. 2 (a) can describe the propagation of a quasi-TEM wave in the slow-mode region. In a straightforward calculation the following difference-differential equations can be derived

$$V_{k+1} - 2V_k + V_{k-1} = L \frac{d^2 Q_k}{dt^2} + R \frac{dQ_k}{dt} \quad (1)$$

and  $\frac{dQ_k}{dt} = G \{ V_k - V(Q_k) \}, \quad (2)$

where  $Q_k$  denotes the charge of the  $k$ -th capacitance.  $Q_k$  is a nonlinear function of the applied voltage  $V$  yielding  $C = dQ/dV = C_0(1 - g(V))$ .  $V(Q_k)$  is the inverse function. This is the first result: The general behavior of the nonlinear wave is completely described by simple equations which on the other

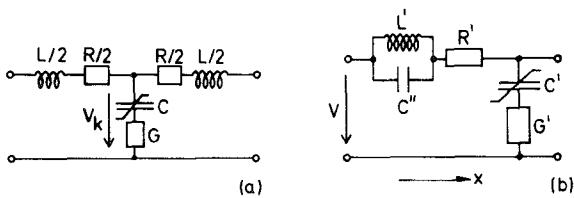


FIG. 2. EQUIVALENT CIRCUITS OF THE PERIODIC STRUCTURE OF FIG. 1 IN THE SLOW-MODE REGION.  
(a) T-ELEMENT WITH LUMPED PARAMETERS.  
(b) EQUIVALENT DISTRIBUTED CIRCUIT.

hand are ideally suited for a direct numerical integration or iteration as well as for further analytical studies. Clearly, applying Floquet's theorem to the linearized equations, the propagation constant is immediately derived. An important consequence of the periodicity is now that the structure in Fig. 1 reveals low pass filter characteristic with the cut-off frequency

$\omega_c = 2(LC_0)^{-1/2}$ , i.e. wave propagation is determined by additional dispersion which is absent in case of the homogeneous transmission line. Provided that the influence of dispersion is negligible the lhs of eq. (1) can be approximated by the usual second order partial differentiation with respect to the space variable.

As a second analytical result the distributed circuit in Fig. 2(b) can equivalently describe the wave propagation within the desired range of frequencies or guide wavelengths. Now  $C''$  accounts for the low pass filter characteristics, i.e. for the dispersion of the line. From Fig. 2(b) the propagation is simply given by

$$\frac{\partial V}{\partial \zeta} = g(V) \frac{\partial V}{\partial \tau} + \kappa \frac{\partial^3 V}{\partial \tau^3} - aV + b \frac{\partial^2 V}{\partial \tau^2} \quad (3)$$

Here  $\zeta = x/2u_0$  and  $\tau = t - x/u_0$  are transformed coordinates,  $u_0 = (L'C_0)^{-1/2}$  is the phase velocity, and  $\kappa = \omega_c^{-2}$  a measure of dispersion.  $a = R'C'$  and  $b = C'_0/G'$  are the lossy parameters. As a special case, now  $\kappa = 0$  is valid when wave propagation along homogeneous transmission lines is considered. Clearly, eq. (3) can particularly be used to describe nonlinear waves along distributed Schottky

diodes.

Equation (3) is a further very interesting result because this kind of wave equation is well-known from applied physics [4]. Equation (3) may be used to describe the generation of harmonics, parametric amplification, self-phase modulation, nonlinear pulse compression, etc.. In the following the second harmonic generation (SHG) with  $g(V) = \delta V$  is discussed in two steps as a quantitative and interesting example for MMIC applications.

#### TRAVELING-WAVE FREQUENCY DOUBLING

The case of small efficiency is studied first. Then for practical purposes the ratio  $v = V_2/V_1$  of the voltage amplitudes of the second harmonic and the fundamental wave can be estimated from

$$v = \delta V_1 \{1 - \exp(-\alpha x)\} \beta/8\alpha \quad (4)$$

where  $\alpha$  is the attenuation and  $\beta$  the phase constant,  $\beta/\alpha$  is a measure of the quality factor. For example, a nonlinearity of  $\delta V_1 = 0.1$  and  $\beta/\alpha = 10$  yield a maximum value of  $v = 12\%$ . Due to dispersion the ratio will even be smaller.

Studying, secondly, the case of large efficiency the losses are assumed to be of second order. In order to achieve a high efficiency the filter characteristic is now essential since it prevents the flow of unwanted harmonics. However, the dispersion causes a phase mismatch between the two waves traveling down the line. Consequently the dispersion must be altered by a suitable structure to get phase synchronism. This is done as outlined in Fig. 3. Here the strip line exhibits an additional periodicity yielding the following properties. In comparison with Fig. 1 every second

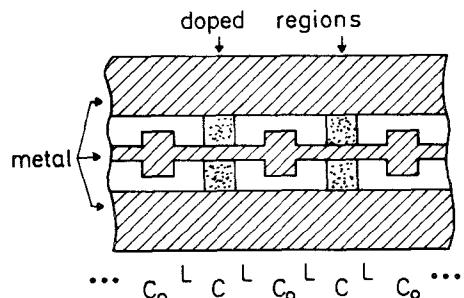


FIG. 3. TOP VIEW OF THE PROPOSED PERIODIC STRUCTURE OF THE SCHOTTKY COPLANAR WAVEGUIDE TO ACHIEVE PHASE MATCHING FOR SHG.

doped layer has been omitted and replaced by the linear capacitance  $C_0$ . The important point is now that the fundamental wave is a forward wave and the second harmonic is a backward wave, so the energy of the second harmonic is available at the input [5].

This concept of a traveling-wave frequency doubler is experimentally verified as follows. The efficiency has been measured using a slow-wave

model structure where a TEM transmission line has periodically been loaded with high quality varactor diodes and linear capacitors according to the arrangement of Fig. 3, cf. /5/. The experimental values are  $L = 23 \text{ nH}$ ,  $C_0 = 9.3 \text{ pF}$ ,  $R = 0.05 \Omega$ ,  $G = 1.3 \Omega^{-1}$ , and  $\delta = 0.12 \text{ V}^{-1}$  leading to a characteristic impedance of  $50 \Omega$ , an attenuation of 0.09 dB per section, and a slowing factor of 5. As a final result, at a frequency of 240 MHz the measured value of efficiency is found to be as high as 95 % using an input power of 80 mW, a length of only 14 sections, and an open ended line. Thus it is concluded that the conversion efficiency of a varactor diode can greatly be enhanced by applying traveling-wave concepts to a periodic chain of coupled elements as described above. Moreover, since the experimental model consists of usual semiconductor devices with relevant transmission line parameters it is estimated that the structure of Fig. 3 can be realized in a monolithic form. This would be the first frequency doubler with an efficiency of nearly 100 % in MMIC technology.

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